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4-D gauged supergravity analysis of type-IIB vacua on $\mathbf{K3} \times \mathbf{T}^2/\mathbb{Z}_2$

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Abstract: We analyze $N = 2, 1, 0$ vacua of type-IIB string theory on $\mathbf{K3} \times \mathbf{T}^2/\mathbb{Z}_2$ orientifold in presence of three-form fluxes from a four dimensional supergravity viewpoint. The quaternionic geometry of the **K3** moduli space together with the special geometry of the NS and R-R dilatons and of the $\mathbf{T}^2$-complex structure moduli play a crucial role in the analysis. The introduction of fluxes corresponds to a particular gauging of $N = 2$, $D = 4$ supergravity. Our results agree with a recent work of Tripathy and Trivedi. The present formulation shows the power of supergravity in the study of effective theories with broken supersymmetry.

Keywords: Supersymmetry Breaking, Supersymmetry and Duality, Supergravity Models.

Contents

1. Introduction

Recently, compactifications of string and M-theories in presence of p-form fluxes have received much attention. They give rise to effective models where the moduli stabilization is a simple consequence of a combined Higgs and super Higgs mechanism [1]–[30].

Interestingly enough, many of these models are examples of no-scale supergravities [31]–[33] in the context of string and M-theory [2] [6]–[10]. Their low energy effective limits can be understood in terms of gauged supergravities, which, for a special choice of the gauging, allow partial supersymmetry breaking without a cosmological term [21, 22] [34]–[36].

A particularly appealing class of examples is obtained by considering type-IIB theory compactified on orientifolds such as $\mathbf{T}^6/\mathbb{Z}_2$ [8, 9, 23, 24, 25] and $\mathbf{K3}\times\mathbf{T}^2/\mathbb{Z}_2$ [30], or Calabi-Yau manifolds [1, 2, 3, 10, 14], in presence of three form fluxes. When compactifiying on the orientifold $\mathbf{T}^6/\mathbb{Z}_2$ with fluxes, one finds vacua with reduced supersymmetry $N = 3, 2, 1, 0$ [8, 9, 23, 24], while in the case of $\mathbf{K3} \times \mathbf{T}^2/\mathbb{Z}_2$, one obtains vacua with $N = 2, 1, 0$ supersymmetry [30].

It is the aim of the present investigation to obtain a gauged supergravity interpretation of the $N = 2, 1, 0$ vacua recently found by Tripathy and Trivedi [30] for the $\mathbf{K3} \times \mathbf{T}^2/\mathbb{Z}_2$ theory. In absence of fluxes we obtain an ungauged $N = 2$ supergravity with a certain

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content of hypermultiplets and vector multiplets [37, 39, 23, 24]. Moreover, the underlying special and quaternionic geometries for these multiplets [37, 38] is determined by the properties of the moduli spaces. The introduction of fluxes is then equivalent to gauge some isometries of the quaternionic manifold by some of the vectors at our disposal in the theory.

$N = 2$ and $N = 1$ vacua stabilize many of the moduli and correspond to two different gaugings: they differ in the choice of quaternionic isometries and in the choice of vectors which realize the gauging.

Let f denote generically the fermions of the theory, and let ϵ be a rigid supersymmetry (constant) parameter in four dimensional Minkowski space. A Poincaré invariant configuration must have all fields equal to zero except for the scalar fields, which can be set to constants. This configuration has an unbroken supersymmetry ϵ if the values of the scalar fields are such that

$$\delta_\epsilon f = 0\,. \tag{1.1}$$

The crucial fact is that these are necessary and sufficient conditions for the configuration to be a supersymmetric vacuum (with unbroken supersymmetry ϵ) with vanishing vacuum energy.

It is our purpose to find solutions to (1.1) in the low energy effective $N = 2$ supergravity derived from compactifications of the type-IIB theory on $\mathbf{K3} \times \mathbf{T}^2/\mathbb{Z}_2$ in presence of fluxes. A similar analysis for the $\mathbf{T}^6/\mathbb{Z}_2$ theory has been done in refs. [23, 24, 25].

2. Type-IIB superstring on a K3 $\times$ $\mathbf{T}^2/\mathbb{Z}_2$ orientifold

Type-IIB compactified on $\mathbf{K3} \times \mathbf{T}^2$ has been widely studied in the context of the type-IIA/type-IIB and type-I/heterotic string dualities [40, 42, 43, 44, 47].

The bulk sector of this theory is largely based on properties of the moduli space [41, 46] of the $\mathbf{K3}$ manifold [47] and the torus $\mathbf{T}^2$. Before the orientifold projection this theory has $N = 4$ supersymmetry. After the $\mathbb{Z}_2$ projection, and in absence of fluxes, the theory has $N = 2$ supersymmetry. We discuss the spectrum of the projected theory [30]. It consists of the following multiplets:

1. the graviton multiplet, $[(2), 2(3/2), (1)]$,
2. three vector multiplets, $3 \times [(1), 2(1/2), 2(0)]$,
3. twenty hypermultiplets, $20 \times [2(1/2), 4(0)]$.

We count first the scalar degrees of freedom remaining after the projection. The internal manifold is parametrized by a pair of complex coordinates on the $\mathbf{K3}$ factor (indexed by $L = 1, 2$) and a pair of real coordinates on the torus (indexed by $i = 1, 2$). The metric of the internal manifold is the direct product metric. The moduli space of the metrics on $\mathbf{K3}$ is, up to a quotient by discrete transformations,

$$\frac{\mathrm{SO}(3,19)}{\mathrm{SO}(3)\times\mathrm{SO}(19)} \times \mathbb{R}^+_{\mathbf{K3}}\,, \tag{2.1}$$

and has dimension 58. We denote by ρ_2 the parameter corresponding to $\mathbb{R}^+_{\mathbf{K3}}$.

The moduli space of the metrics on $\mathbf{T}^2$ is

$$\frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \mathbb{R}^+_{\mathbf{T}^2}\,,$$

of dimension 3 and is parametrized by the Kähler modulus ϕ and the complex structure τ:

$$\phi = \sqrt{g} = \sqrt{g_{11}g_{22} - {g_{12}}^2} \qquad 1 \text{ real scalar} \tag{2.2}$$

$$\tau = \tau_1 + i\tau_2\,, \quad \tau_1 = \frac{g_{12}}{g_{22}}\,, \quad \tau_2 = \frac{\sqrt{g}}{g_{22}}\,, \qquad 1 \text{ complex scalar}\,. \tag{2.3}$$

Let $C_{\mu\nu\rho\sigma}$ be the four-form field of type-IIB, with self-dual field strength. There are 23 RR real scalars that come from this form when the indices are taken along the internal manifold. The massless modes correspond to cohomology classes. The Hodge numbers of odd order on **K3** are zero. For even order, the only non vanishing ones are $h_{0,0} = h_{2,2} = 1$, $h_{2,0} = h_{0,2} = 1$ and $h_{1,1} = 20$. The torus $\mathbf{T}^2$ has Betti numbers $b_2 = b_0 = 1$ and $b_1 = 2$. Then, the components of $C_{\mu\nu\rho\sigma}$ that will give rise to scalar fields are of the following forms:

$$C_{LMij} = C\epsilon_{LM}\epsilon_{ij}\,, \qquad \text{one complex scalar} \tag{2.4}$$

$$C_{L\bar{M}ij} = C_{L\bar{M}}\epsilon_{ij}\,, \qquad 20 \text{ real scalars} \tag{2.5}$$

$$C_{L\bar{P}M\bar{Q}} = \rho_1\epsilon_{LM}\epsilon_{\bar{P}\bar{Q}}\,, \qquad 1 \text{ real scalar}\,. \tag{2.6}$$

Finally, we have the two type-IIB dilatons that give two scalars in four dimensions. We denote them by φ_0 and C_0.

The manifold of the **K3** metrics (2.1), with $\mathbb{R}^+_{\mathbf{K3}}$ replaced by $\phi = \mathbb{R}^+_{\mathbf{T}^2}$ (see below for an explanation), enlarges with the 22 scalars of (2.4) and (2.5) to the quaternionic manifold [41]

$$\frac{\mathrm{SO}(4,20)}{\mathrm{SO}(4) \times \mathrm{SO}(20)}\,.$$

This manifold has real dimension 80, corresponding to the 80 scalars of the twenty hypermultiplets.

To understand the assignment of the $\mathbb{R}^+$ factors to the scalar manifolds, let us look at the kinetic term for the two-form field strengths as they come from ten dimensions (up to a factor depending on the dilaton),

$$\sqrt{g_{10}}\, g^{\hat{\mu}_1\hat{\nu}_1} g^{\hat{\mu}_2\hat{\nu}_2} g^{\hat{\mu}_3\hat{\nu}_3} H_{\hat{\mu}_1\hat{\mu}_2\hat{\mu}_3} H_{\hat{\nu}_1\hat{\nu}_2\hat{\nu}_3}\,.$$

When compactifying on $\mathbf{K3} \times \mathbf{T}^2$, the volume factorizes as $\sqrt{g_{10}} = \sqrt{g_4}\sqrt{g_2}\nu$, where ν is the volume on **K3** and $\sqrt{g_2}$ the volume on $\mathbf{T}^2$. Since the bulk vectors in $D = 4$ arise by taking an index along $\mathbf{T}^2$, the relevant term is

$$\sqrt{g_4}\sqrt{g_2}\nu g^{\mu_1\nu_1} g^{\mu_2\nu_2} g^{ij} H_{\mu_1\mu_2 i} H_{\nu_1\nu_2 j}\,.$$

The factor $\sqrt{g_2}g^{ij}$ is conformally invariant in dimension 2 and depends only on the complex structure of the torus τ, while the modulus $\nu = \rho_2$ appears explicitly. Also, if one includes D7 brane gauge fields, their four dimensional coupling depends on the **K3** volume but not on the $\mathbf{T}^2$ Kähler modulus.[1] Then the coordinate ρ_2 of the **K3** volume sits in a vector multiplet. These couplings are insensitive to the rescaling of the Einstein-Hilbert term.

[1]We acknowledge an enlightening conversation with C. Angelantonj on this point.

The three complex scalars of the vector multiplets are

$$\begin{aligned}\rho =&\ \rho_1 - i\rho_2\,, \qquad \mathrm{Im}\,\rho < 0\\ \tau =&\ \tau_1 + i\tau_2\,, \qquad \mathrm{Im}\,\tau > 0\\ \sigma =&\ C_0 + ie^{\varphi_0}\,, \qquad \mathrm{Im}\,\sigma > 0\end{aligned} \tag{2.7}$$

parametrizing the coset

$$\frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)}\,.$$

The fact that the $\mathbf{T}^2$ complex structure moduli are in a vector multiplet can also be understood by considering that type-IIB on the orientifold $\mathbf{K3} \times \mathbf{T}^2/\mathbb{Z}_2$ is a truncation to $N = 2$ of $N = 4$ supergravity corresponding to the compactification of type IIB on $\mathbf{K3} \times \mathbf{T}^2$, whose moduli space is

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(6,22)}{\mathrm{SO}(6) \times \mathrm{SO}(22)}\,.$$

There, the complex structure moduli are in the factor $\mathrm{SU}(1,1)/\mathrm{U}(1)$, which is a Kähler-Hodge manifold [44].

2.1 The quaternionic manifold $\mathrm{SO}(4,20)/\mathrm{SO}(4) \times \mathrm{SO}(20)$ as a fibration over $\mathrm{SO}(3,19)/\mathrm{SO}(3) \times \mathrm{SO}(19)$

The coset $\mathcal{M}_q = \mathrm{SO}(4,20)/\mathrm{SO}(4) \times \mathrm{SO}(20)$ is the symmetric space associated to the Cartan decomposition

$$\mathfrak{so}(4,20) = \mathfrak{h} + \mathfrak{p}\,, \qquad \text{with} \qquad \mathfrak{h} = \mathfrak{so}(4) + \mathfrak{so}(20) \qquad \text{and} \qquad \mathfrak{p} = (\mathbf{4},\mathbf{20})\,.$$

We want to give a local parametrization of this coset which displays $\mathcal{N} = \mathrm{SO}(3,19)/(\mathrm{SO}(3)\times \times \mathrm{SO}(19))$ as a submanifold of $\mathcal{M}_q$. It can indeed be proven that globally, $\mathcal{M}_q$ is a fibration over $\mathcal{N}$ [48].

Consider the following decomposition of the Lie algebra $\mathfrak{so}(4,20)$:

$$\mathfrak{so}(4,20) = \mathfrak{so}(3,19) + \mathfrak{so}(1,1) + (\mathbf{3},\mathbf{19})^+ + (\mathbf{3},\mathbf{19})^-\,. \tag{2.8}$$

According to this decomposition, we can find a local parametrization of $\mathcal{M}_q$ via the following element (coset representative) of $\mathrm{SO}(4,20)$:

$$G = \mathrm{e}^{C^I Z_I}\mathrm{e}^{\phi S} L\,, \tag{2.9}$$

where $\{Z_I\}_{I=1}^{22}$ is a set of generators of the abelian subalgebra $(\mathbf{3},\mathbf{19})^+$ in (2.8), S is the generator of $\mathfrak{so}(1,1)$ and L is a coset representative of $\mathcal{N}$, $L \in \mathrm{SO}(3,19)$. It is given in terms of 57 parameters e_m^a, $m = 1,2,3$, $a = 1,\ldots,19$ as

$$L = \begin{pmatrix} (1+ee^T)^{\frac{1}{2}} & e \\ e^T & (1+e^Te)^{\frac{1}{2}} \end{pmatrix}.$$

Because of the action of $\mathrm{SO}(4,20)$, there is a submanifold parametrized by the coordinates C^I with the topology of $S^3 \times S^{19}/\mathbb{Z}_2$.

In this parametrization, the Maurer-Cartan form is simply

$$G^{-1}dG = \mathrm{e}^{\phi}(L^{-1})^{I}_{J}dC^{J}Z_{I} + d\phi S + L^{-1}dL\,. \tag{2.10}$$

The connection and the vielbein 1-forms are the projections of the Maurer-Cartan form over the spaces $\mathfrak{h}$ and $\mathfrak{p}$ respectively. In fact, since the fundamental representation of $\mathrm{SO}(n,m)$ is real, these projections correspond to the symmetric and antisymmetric parts of the matrices,

$$\begin{aligned}\left(G^{-1}dG\right)_{\text{antisym}} &= \left(G^{-1}dG\right)_{\mathfrak{h}} \text{ is the connection 1-form}\,,\\ \left(G^{-1}dG\right)_{\text{sym}} &= \left(G^{-1}dG\right)_{\mathfrak{p}} \text{ is the vielbein 1-form}\,.\end{aligned}$$

We want to write explicitly the vielbein one-form. We take G in the fundamental representation of $\mathrm{SO}(4,20)$, $G(q)^{\Lambda}_{\Sigma}$, with q^u the coordinates on the quaternionic manifold. Since $G(q)^{\Lambda}_{\Sigma}$ is a coset representative, we will be interested in the transformation properties to the right with respect to the subgroup $H = \mathrm{SO}(4)\times\mathrm{SO}(20)$, and will denote it as $G(q)^{\Lambda}_{\sigma}$. The Maurer-Cartan form will have indices in the same representation of $\mathfrak{h}$,

$$\left(G^{-1}dG\right)^{\lambda}_{\sigma}\,, \qquad \lambda,\sigma = 1\,,\ldots,24\,.$$

Notice that $H = \mathrm{SU}(2)_R \times \mathrm{SU}(2) \times \mathrm{SO}(20) \subset \mathrm{SU}(2)_R \times \mathrm{USp}(40)$, where $\mathrm{SU}(2)_R$ is the R-symmetry group and $\mathrm{SU}(2)_R \times \mathrm{USp}(40)$ is the holonomy group of a general quaternionic manifold.

Since we are interested in showing explicitly the symmetry under $\mathrm{SO}(3,19)$, we decompose

$$(\mathbf{4}+\mathbf{20}) \xrightarrow[\mathrm{SO}(3)\times\mathrm{SO}(19)]{} (\mathbf{3}+\mathbf{19})+\mathbf{1}+\mathbf{1}\,.$$

In this decomposition, the diagonal group $(\mathrm{SU}(2)_R\times\mathrm{SU}(2))_{\text{diag}}$ in H is identified with the first factor $\mathrm{SO}(3)$.

For the vielbein we have,

$$(G^{-1}dG)_{\text{sym}} = U_u dq^u = \begin{pmatrix} 0 & P^{m}_{b} & U^{m} & 0\\ (P^T)^{a}_{n} & 0 & 0 & V^{a}\\ (U^T)_{n} & 0 & 0 & d\phi\\ 0 & {V^T}_{b} & d\phi & 0\end{pmatrix}, \tag{2.11}$$

with $P^m_b = \left(L^{-1}dL\right)^{m}_{a}$ the vielbein of the scalar manifold $\mathrm{SO}(3,19)/(\mathrm{SO}(3)\times\mathrm{SO}(19))$ and

$$U^m = \mathrm{e}^{\phi}\left[\left[\left(\mathbb{1}+e\cdot e^T\right)^{\frac{1}{2}}\right]^{m}_{n} dC^n + e^{m}_{a}dC^a\right] \tag{2.12}$$

$$V^a = \mathrm{e}^{\phi}\left[e^{a}_{m}dC^m + \left[\left(\mathbb{1}+e^T\cdot e\right)^{\frac{1}{2}}\right]^{a}_{b} dC^b\right]. \tag{2.13}$$

Notice that the space $\mathfrak{p}$ corresponds to the $(\mathbf{4},\mathbf{20})$ representation of H, which decomposes as

$$(\mathbf{4},\mathbf{20}) \xrightarrow[\mathrm{SO}(3)\times\mathrm{SO}(19)]{} (\mathbf{3},\mathbf{19})+(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{19})+(\mathbf{1},\mathbf{1})$$

and each block in (2.11) corresponds to one of these.

We note that the components on the generators Z_I of the Maurer-Cartan form

$$\mathrm{e}^{\phi}\left(L^{-1}\right)^{I}_{\ J} dC^J$$

contribute both to the vielbein and to the connection of the quaternionic manifold $\mathcal{M}$. In particular, the contribution of this term to the $\mathrm{SU}(2)_R$-connection is proportional to

$$\omega^x_I dC^I \propto \mathrm{e}^{\phi}\left(L^{-1}\right)^{x}_{\ I} dC^I = \mathrm{e}^{\phi}\left(1 + ee^T\right)^{x}_{\ m} dC^m - \mathrm{e}^{\phi} e^x_a dC^a\,, \qquad x = 1,2,3\,. \tag{2.14}$$

These formulae will be useful in the calculation of the scalar potential.

2.2 Vector multiplets and special geometry

The vector multiplet moduli space is

$$\mathcal{M}_{n_v=3} = \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)}\,.$$

It is a special Kähler-Hodge manifold of the series [33, 45]

$$\mathcal{M}_{n_v} = \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SO}(2,n_v-1)}{\mathrm{SO}(2)\times\mathrm{SO}(n_v-1)}$$

with $n_v = 3$.

Let $\mathcal{L}$ be the Hodge line bundle. The special geometry [49, 50, 51] of $\mathcal{M}_{n_v}$ consists on a holomorphic vector bundle $\mathcal{H}$ with structure group $\mathrm{Sp}(2+2n_v,\mathbb{R})$ and a global section

$$\Omega = \begin{pmatrix} X^\Lambda(z) \\ F_\Lambda(z) \end{pmatrix}$$

on $\mathcal{H}\otimes\mathcal{L}$ such that the Kähler form J is given by

$$J = -\frac{i}{2\pi}\partial\bar\partial \ln i\left[\bar X^\Lambda F_\Lambda - \bar F_\Lambda X^\Lambda\right].$$

In an open set the Kähler potential is given by

$$K = -\ln i\left[\bar X^\Lambda F_\Lambda - \bar F_\Lambda X^\Lambda\right].$$

In a point z of the intersection of two open sets the section transforms as

$$\begin{pmatrix} X^\Lambda(z) \\ F_\Lambda(z) \end{pmatrix} = \mathrm{e}^{f(z,A,B,C,D)}\begin{pmatrix} A^\Lambda_{\ \Lambda'} & B^{\Lambda\Lambda'} \\ C_{\Lambda\Lambda'} & D_\Lambda^{\ \Lambda'} \end{pmatrix}\begin{pmatrix} X^{\Lambda'}(z) \\ F_{\Lambda'}(z) \end{pmatrix},$$

where (A,B,C,D) define a constant, symplectic transformation:

$$A^T D - C^T B = \mathbb{1}\,, \qquad A^T C = C^T A\,, \qquad B^T D = D^T B\,,$$

and $f(z,A,B,C,D)$ is a holomorphic phase of the Hodge bundle.

From the doublet of two-forms in IIB, $B^{\alpha}_{\mu\nu}$, one can obtain vector fields in four dimensions when one of the indices is taken over the torus $\mathbf{T}^2$ (the odd cohomology of $\mathbf{K3}$ is zero). Then the four gauge fields in four dimensions are [30]

$$A^{\alpha i}_{\mu} = B^{\alpha}_{\mu i}\,.$$

Therefore they are in the representation $(1/2, 1/2)$ of the type-IIB R-symmetry $\mathrm{SO}(2,1) \simeq \mathrm{SL}(2,\mathbb{R})$, times the $\mathrm{SL}(2,\mathbb{R})$ associated to the $\mathbf{T}^2$ complex structure. In the homomorphism $\mathrm{SO}(2,1) \times \mathrm{SO}(2,1) \simeq \mathrm{SO}(2,2)$ the $(1/2,1/2)$ goes to the fundamental representation, so we can set an index $\Lambda = (i,\alpha) = 0,1,2,3$ and denote the gauge fields as A^{Λ}_{μ}.

We have therefore to choose a symplectic embedding of $\mathrm{SO}(2,2) \times \mathrm{SL}(2,\mathbb{R})$ in $\mathrm{Sp}(8,R)$ such that $\mathrm{SO}(2,2)$ is an electric subgroup. The third $\mathrm{SL}(2,\mathbb{R})$ instead acts on the vectors as an electric-magnetic duality.

Let $\eta = \mathrm{diag}(+1,+1,-1,-1)$. $\mathrm{SO}(2,2)$ is embedded in $\mathrm{Sp}(8,R)$ as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}, \qquad A \in \mathrm{SO}(2,2)\,, \qquad D = \left(A^T\right)^{-1}\,, \qquad B = C = 0\,,$$

while the third $\mathrm{SL}(2,\mathbb{R})$ is embedded as

$$\begin{pmatrix} a\mathbb{1} & b\eta \\ c\eta & d\mathbb{1} \end{pmatrix}, \qquad ad - bc = 1\,.$$

In order to have the symplectic embedding chosen above explicitly manifest in the theory, one has to choose a local frame for the symplectic bundle. In this frame, the global section Ω of the special geometry is given by [52]:

$$\Omega = \left(X^{\Lambda}(\sigma,\tau), F_{\Lambda} = \rho\eta_{\Lambda\Sigma}X^{\Sigma}(\sigma,\tau)\right)\,,$$

with

$$X^{\Lambda}X^{\Sigma}\eta_{\Lambda\Sigma} = 0\,, \qquad X^{\Lambda}\bar{X}^{\Sigma}\eta_{\Lambda\Sigma} = \mathrm{e}^{-\hat{K}}\,,$$

and where $\hat{K} = \frac{1}{2}i(\bar{\tau} - \tau)i(\bar{\sigma} - \sigma)$ is the Kähler potential of the submanifold $\mathrm{SO}(2,2)/(\mathrm{SO}(2) \times \mathrm{SO}(2))$. The explicit dependence of X^{Λ} in terms of the local coordinates (ρ,σ,τ) is

$$X^0 = \frac{1}{2}(1 - \sigma\tau)\,, \quad X^1 = -\frac{1}{2}(\sigma + \tau)\,, \quad X^2 = -\frac{1}{2}(1 + \sigma\tau)\,, \quad X^3 = \frac{1}{2}(\tau - \sigma)\,. \tag{2.15}$$

It is important to notice that in this embedding F_{Λ} cannot be written as $\partial_{\Lambda}F(X)$. The prepotential $F(X)$ does not exist. This allows to have partial breaking of $N = 2$ supersymmetry [35], otherwise impossible [34].

The Kähler potential of $\mathcal{M}_{n_v=3}$ is given by the formula

$$\mathrm{e}^{-K} = i\left(\bar{X}^{\Lambda}F_{\Lambda} - X^{\Lambda}\bar{F}_{\Lambda}\right) = \frac{i}{2}(\rho - \bar{\rho})i(\bar{\tau} - \tau)i(\bar{\sigma} - \sigma) = \mathrm{e}^{-\tilde{K}}\mathrm{e}^{-\hat{K}}\,, \tag{2.16}$$

where $\mathrm{e}^{-\tilde{K}} = i(\rho - \bar{\rho})$.

From the symplectic section we get the following kinetic matrix for the vectors [52, 39]

$$\mathcal{N}_{\Lambda\Sigma} = (\rho - \bar{\rho})(\phi_\Lambda \bar{\phi}_\Sigma + \bar{\phi}_\Lambda \phi_\Sigma) + \bar{\rho}\eta_{\Lambda\Sigma} , \qquad \phi^\Lambda = \frac{X^\Lambda}{(X^\Sigma \bar{X}_\Sigma)^{\frac{1}{2}}} ,$$

which has the important property

$$-\frac{1}{2}(\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma})^{-1} - 4\bar{L}^{(\Lambda}L^{\Sigma)} = -\frac{\eta^{\Lambda\Sigma}}{i(\rho - \bar{\rho})} .$$

3. Gauging of the traslational isometries and fluxes

The gauging of the present theory involves the four abelian vectors and the scalars of the quaternionic manifold, in particular the axions of the 22 abelian isometries, whose Killing vectors correspond in the Lie algebra (2.8) to a lorentzian vector of SO(3, 19). We can in principle gauge a four dimensional subalgebra of this abelian algebra. As in (2.9) we denote by (C_m, C_a) with $m = 1, 2, 3$ and $a = 1, \ldots, 19$, the 22 axions. Their covariant derivatives are

$$\begin{aligned} D_\mu C_m &= \partial_\mu C_m + f_{m,\Lambda} A^\Lambda_\mu , \\ D_\mu C_a &= \partial_\mu C_a + h_{a,\Lambda} A^\Lambda_\mu , \end{aligned}$$

with $\Lambda = 0, \ldots, 3$. $f_{m,\Lambda}$, $h_{a,\Lambda}$ are the coupling constants. When performing the dimensional reduction, the kinetic terms for the axions appear with these covariant derivatives, and the coupling constants are related to the three-form fluxes on $\mathbf{K3} \times \mathbf{T}^2$.

More precisely, let us undo the relabeling of the SO(2, 2) vector index, $\Lambda = (i, \alpha)$, $i = 1, 2$ and $\alpha = 1, 2$ as in section 2.2, and the 22 vector indices $(m, a) = (LM, L\bar{M})$ as in (2.4), (2.5). Then the coupling constants become

$$(\tilde{f}_{i,LM,\alpha}, \tilde{h}_{i,L\bar{M},\alpha}) .$$

They can be identified with the three-form fluxes with one index on $\mathbf{T}^2$ and the other two on $\mathbf{K3}$.

There are different choices for the coupling constants, according to the supersymmetries that we want to have for the vacua:

- For configurations with $N = 2$ supersymmetry we will take $f_{m,\Lambda} = 0$, $h_{a,0} = h_{a,1} = 0$ and $h_{1,2} = g_2$, $h_{2,3} = g_3$. The vectors that are "higgsed" (that acquire mass) are the vector partners of the IIB dilaton and the $\mathbf{T}^2$ complex structure moduli.
- For configurations with $N = 1, 0$ supersymmetry we will take, in the first place, $h_{a,\Lambda} = 0$, $f_{m,2} = f_{m,3} = 0$ and only $f_{1,0} = g_0$, $f_{2,1} = g_1$ different from zero. In this case, the vectors that acquire mass are the graviphoton and the vector partner of the $\mathbf{K3}$ volume modulus.

This means that the coupling constant SO(2, 2)-vector g_Λ (with metric diag(+, +, −, −)) has negative norm for $N = 2$ configurations and positive norm for $N = 0, 1$ configurations. The $N = 1$ is obtained by imposing the further constraint, $|g_0| = |g_1|$.

- For $N=2$ supersymmetry preserving vacua, no other choices are allowed, while for configurations with $N=1,0$ supersymmetry there exists a more general choice, with all couplings g_Λ non vanishing. All the vectors acquire mass. These configurations will be discussed separately in section 5.1.

4. Supersymmetric vacua

As a consequence of the supersymmetric Ward identities [53] one can obtain any supersymmetric configuration as a solution of the constraints

$$\delta_\epsilon \lambda = 0\,,$$

where ϵ is the constant parameter of the global unbroken supersymmetry and λ are the spin 1/2 fields. This can be done without looking to the explicit form of the potential. If we look for vacua with a Poincaré symmetry, one has the further constraint

$$\delta_\epsilon \psi_{\mu A} = 0\,,$$

with $\psi_{\mu A}$ the two gravitino fields.

In $N=2$ supergravity, we have hypermultiplets and vector multiplets. The fermion fields denote always the chiral projections. We denote by $\psi_{\mu A}$ the gravitinos, with $A=1,2$ refering to the SU(2) R-symmetry; by λ^{iA} the gauginos with $i=1,\dots,n_v$, which form a contravariant vector on the special Kähler manifold $\mathcal{M}_{n_v}$; and the hyperinos by ζ_α, with $\alpha=1,\dots,2n_h$ (n_h is the number of hypermultiplets). The index alpha is a vector index of USp$(2n_h)$, which together with SU(2) (index A=1,2) form the reduced holonomy of the quaternionic manifold.

We will denote by $k_\Lambda = k^u_\Lambda \partial_u$ the Killing vectors of the translational isometries of the quaternionic manifold, which will be gauged (hence with an index $\Lambda=0,\dots,3$ as the vectors). Their prepotential is denoted by P^x_Λ, with $x=1,2,3$ (it is an SU(2) triplet). If Ω_{uv} is the curvature two form, and Ω^x_{uv} its $\mathfrak{su}(2)$ components, then

$$k_{u\Lambda} = -\frac{1}{6}\Omega^x_{uv}\nabla^v P^x_\Lambda\,.$$

(The index x is contracted with the euclidean metric).

In addition, we denote by $g_{i\bar{j}}$ the Kähler metric of the special manifold $\mathcal{M}_{n_v}$, K the Kähler potential. In terms of the holomorphic section Ω we can define

$$V = \mathrm{e}^{K/2}\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}$$

which is not holomorphic but is *covariantly holomorphic*

$$\mathcal{D}_{\bar{i}} V = (\partial_{\bar{i}} - \frac{1}{2}\partial_{\bar{i}} K)V = 0\,.$$

The supersymmetry transformations of the fermionic fields for a constant parameter ϵ are as follows [54]:

$$\delta\psi_{A\mu} = -\frac{1}{2}P^x_\Lambda X^\Lambda \mathrm{e}^{\frac{K}{2}}(\sigma^x)_{AB}\gamma_\mu\epsilon^B \tag{4.1}$$

$$\delta\lambda^{iA} = ig^{i\bar{j}}\mathcal{D}_{\bar{j}}(\bar{X}^\Lambda \mathrm{e}^{\frac{K}{2}})P^x_\Lambda(\sigma^x)^{AB}\epsilon_B \tag{4.2}$$

$$\delta\zeta_{\tilde{A}a} = 2\varepsilon^{AB}U_{A\tilde{A}a,u}k^u_\Lambda\bar{X}^\Lambda \mathrm{e}^{\frac{K}{2}}\epsilon_B \tag{4.3}$$

$$\delta\zeta_{\tilde{A}} = 2\varepsilon^{AB}U_{A\tilde{A},u}k^u_\Lambda\bar{X}^\Lambda \mathrm{e}^{\frac{K}{2}}\epsilon_B \tag{4.4}$$

where the hyperinos transformation laws are decomposed with respect to the manifest holonomy SU(2) $\times$ SO(19) (so that $\alpha \to (\tilde{A}a, \tilde{A})$; $\tilde{A} = 1, 2$), and

$$U_{A\tilde{A}a,u} = \varepsilon_{A\tilde{A}}V_{a,u}\,, \qquad U_{A\tilde{A},u} = (\sigma^m)_{A\tilde{A}}U_{m,u}$$

with U_m, V_a given in (2.11).

4.1 N=2 supersymmetric configurations

To have $N = 2$ configurations with vanishing vacuum energy, the variations of all fermions must vanish for any constant supersymmetry parameter ϵ_A ($A = 1, 2$). This demands, from equations (4.1) and (4.2), that

$$P^x_\Lambda = 0 \qquad x = 1, 2, 3\,, \quad \Lambda = 0, 1, 2, 3 \tag{4.5}$$

and, from the hyperinos variations (4.3), (4.4), that

$$k^u_\Lambda X^\Lambda = 0\,, \qquad u = 1, \ldots, 80\,. \tag{4.6}$$

To preserve $N = 2$ supersymmetry, the graviphoton A^0_μ cannot acquire mass. We can switch on interactions for A^Λ_μ, with $\Lambda = 2, 3$, gauging two of the isometries associated to the 19 axions C^a. We take the two Killing vectors k^u_2 and k^u_3 whose only non vanishing components are

$$\begin{aligned} k^u_2 &= g_2 \neq 0 \quad \text{for } q^u = C^{a=1}\,, \quad \text{and} \\ k^u_3 &= g_3 \neq 0 \quad \text{for } q^u = C^{a=2}\,, \end{aligned}$$

for arbitrary constants g_2 and g_3.

Inserting in equation (4.6), this implies

$$X^2(\sigma, \tau) = X^3(\sigma, \tau) = 0 \tag{4.7}$$

Equation (4.7) stabilizes the two vector-multiplets moduli (σ, τ), since from (2.15) we have:

$$\tau = \sigma\,, \sigma^2 = -1 \quad \Longrightarrow \quad \sigma = \tau = i\,.$$

Let us note that to have $N = 2$ preserving vacua it is not possible to gauge more than two vectors, since it would give extra constraints on the X^Λ, incompatible with (4.7) in the given symplectic frame (2.15). Also, the two vectors that realize the gauging have to be A^2_μ and A^3_μ, which are in the same multiplets as the coordinates σ and τ. Indeed, it is easy to see that this choice is the only one stabilizing the moduli compatibly with the conditions $\mathrm{Im}(\sigma), \mathrm{Im}(\tau) > 0$ of (2.7). This same result is obtained in ref. [30], section 5 with topological arguments.

Equation (4.5) is solved by recalling that, for gauged axion symmetries [11, 14], the expression for the prepotential P^x_Λ gets simplified to

$$P^x_\Lambda = \omega^x_u k^u_\Lambda \,,$$

where ω^x_u is the $\mathrm{SU}(2)_R$-connection. In this case we have

$$P^x_\Lambda = \sum_{a=1}^{2} \omega^x_a k^a_\Lambda = \sum_{a=1}^{2} \mathrm{e}^{\phi} e^x_a k^a_\Lambda \,, \qquad \text{for } \Lambda = 2, 3$$

where we have used equation (2.14).

$P^x_\Lambda = 0$ then implies

$$e^x_a = 0 \qquad \text{for } a = 1, 2 \,.$$

The C^a, $a = 1, 2$ are Goldstone bosons. They disappear form the spectrum, making massive the gauge vectors A_2 and A_3. In fact, two of the original massless hypermultiplets (corresponding to the degrees of freedom C^a and e^x_a for $a = 1, 2$) and the two vector multiplets of A_2 and A_3, combine into two long massive vector multiplets $[1, 4(\frac{1}{2}), 5(0)]$.

We see that the $N = 2$ configurations are just an example of the Higgs phenomenon of two vector multiplets. The residual moduli space is

$$\frac{\mathrm{SO}(4, 18)}{\mathrm{SO}(4) \times \mathrm{SO}(18)} \times \frac{\mathrm{SU}(1, 1)}{\mathrm{U}(1)} \,.$$

The $\mathrm{SU}(1, 1)/\mathrm{U}(1)$ factor contains the **K3** volume modulus, pertaining to the remaining massless vector multiplet. The moduli corresponding to the **K3** metrics form the submanifold

$$\frac{\mathrm{SO}(3, 17)}{\mathrm{SO}(3) \times \mathrm{SO}(17)} \times \mathbb{R} \,,$$

in accordance with ref. [30].

4.2 N=1 supersymmetric configurations

In the $N = 1$ supersymmetric vacua, the graviphoton and the vector partner of **K3** volume modulus are gauged. Indeed, in any truncation of $N = 2 \to N = 1$ supergravity with Poincaré vacuum, the graviphoton must become massive [55]. The charge vector g^Λ (in the notation of section 3) can be chosen with components $f_{1,0} = g^0 \neq 0$, $f_{2,1} = g^1 \neq 0$ and the rest zero. This means that we switch on the charges of the isometries associated to two of the three axions C^m.

The relevant Killing vectors are

$$\begin{aligned} k^u_0 &= g_0 \neq 0 \quad \text{for} \quad q^u = C^{m=1} \\ k^u_1 &= g_1 \neq 0 \qquad \text{for} \quad q^u = C^{m=2} \,. \end{aligned}$$

The quaternionic prepotential for constant Killing vectors is

$$P^x_\Lambda = \omega^x_u k^u_\Lambda \,, \qquad \Lambda = 0, 1 \,,$$

so

$$P^x_0 = \omega^x_{C_1} g_0 \,, \qquad P^x_1 = \omega^x_{C_2} g_1 \,.$$

Using equation (2.14), we have

$$P^x_0 = \mathrm{e}^\phi (1 + e e^t)^{\frac{1}{2}}{}^x{}_1 g_0 \,, \qquad P^x_1 = \mathrm{e}^\phi (1 + e e^t)^{\frac{1}{2}}{}^x{}_2 g_1 \,.$$

We want to study vacua that preserve one supersymmetry (we choose ϵ_2). We have to impose that $\delta_{\epsilon_2} f = 0$ in equations (4.1)–(4.4).

From the variation of the antichiral hyperinos we have (4.3), (4.4)

$$\begin{aligned}
\delta_{\epsilon_2} \zeta^{\tilde A a} &= 0 \Longrightarrow V^a_m k^m_\Lambda X^\Lambda = 0 \,, \qquad \Lambda = 0, 1 \,, \quad m = 1, 2 \,, \\
\delta_{\epsilon_2} \zeta^{\tilde A} &= 0 \Longrightarrow \sigma^{m \tilde A}{}_B U_{m\,n} k^n_\Lambda X^\Lambda = 0, \quad \Lambda = 0, 1 \,, \; m = 1, 2, 3, \; n = 1, 2 \,.
\end{aligned}$$

The first of these equations implies

$$e^a_m = 0 \,, \quad m = 1, 2 \,, \tag{4.8}$$

and the second one turns out to be proportional to the equation for the variation of the gravitino. We solve it below.

From the variation of the gravitino (4.1) we obtain

$$\begin{aligned}
\delta_{\epsilon_2} \psi_{1\mu} &= 0 \Longrightarrow S_{12} \propto P^x_\Lambda X^\Lambda \sigma^x_{1\,2} = 0 \,, \\
\delta_{\epsilon_2} \psi_{2\mu} &= 0 \Longrightarrow S_{22} \propto P^x_\Lambda X^\Lambda \sigma^x_{2\,2} = 0 \,,
\end{aligned}$$

in terms of the mass matrix of the gravitinos:[2]

$$S_{AB} = \frac{i}{2} P^x_\Lambda X^\Lambda \mathrm{e}^{\frac{K}{2}} (\sigma^x)_{AB} = \frac{i}{2} \mathrm{e}^{\frac{K}{2}} \left(P^x_0 \sigma^x_{AB} X^0 + P^x_1 \sigma^x_{AB} X^1 \right) .$$

Using (4.8), S_{AB} becomes proportional to

$$\mathrm{e}^\phi \mathrm{e}^{\frac{K}{2}} \begin{pmatrix} g_0 X^0 + i g_1 X^1 & 0 \\ 0 & -g_0 X^0 + i g_1 X^1 \end{pmatrix} , \tag{4.9}$$

and an $N = 1$ invariant vacuum requires

$$S_{22} \propto -g_0 X^0 + i g_1 X^1 = 0 \,. \tag{4.10}$$

Finally, from the (antichiral) gauginos variation we find

$$\delta_{\epsilon_2} (\lambda)^{\bar i}_A = 0 \Longrightarrow P^x_\Lambda \mathcal{D}_j \left(X^\Lambda \mathrm{e}^{\frac{K}{2}} \right) (\sigma^x)_{A2} = 0 \,.$$

[2] The sigma matrices with the two indices down are $(\sigma_3, i\mathbb{1}, \sigma_1)$.

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(Notice that we have used the complex conjugate of the chiral gaugino λ^{iA}). The relevant matrix is

$$\mathrm{e}^{\phi}\begin{pmatrix} g_0\mathcal{D}_i(X^0\mathrm{e}^{\frac{K}{2}})+ig_1\mathcal{D}_i(X^1\mathrm{e}^{\frac{K}{2}}) & 0 \\ 0 & -g_0\mathcal{D}_i(X^0\mathrm{e}^{\frac{K}{2}})+ig_1\mathcal{D}_i(X^1\mathrm{e}^{\frac{K}{2}}) \end{pmatrix},$$

The second term in the covariant derivative $\mathcal{D}_i=(\partial_i+\frac{1}{2}\partial_i K)$ gives a contribution proportional to (4.9). The first term together with (4.10) gives the conditions

$$\begin{aligned} -g_0\partial_\sigma X^0+ig_1\partial_\sigma X^1 &= 0\,, \\ -g_0\partial_\tau X^0+ig_1\partial_\tau X^1 &= 0\,, \end{aligned}$$

which imply

$$\tau=\sigma=i\frac{g_1}{g_0}\,.$$

(Note that $\partial_\rho X^\Lambda=0$, so there are no further constraints.) Then equation (4.10) gives

$$-g_0\left(1-\sigma^2\right)-2ig_1\sigma=0\,,$$

and by using $\sigma=ig_1/g_2$ we get

$$g_0^2=g_1^2\,,$$

which implies $\tau=\sigma=i$ and $g_0=g_1$ (the other possibility $g_0=-g_1$ would give σ and τ outside their domain of definition.) Note that at this point $X^2=X^3=0$ as in the $N=2$ case. Also, this point is the self dual point of $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ (fixed point of the transformation $\tau\to-1/\tau$).

We now summarize the massless spectrum of the $N=1$ reduced theory. From the 58 scalars of $\mathrm{SO}(3,19)/(\mathrm{SO}(3)\times\mathrm{SO}(19))\times\mathbb{R}^+_{\mathbf{T}^2}$ there remain 20 scalars parametrizing $\mathrm{SO}(1,19)/\mathrm{SO}(19)\times\mathbb{R}^+_{\mathbf{T}^2}$. From the 22 axions there remain 20. All together they complete the scalars of 20 chiral multiplets. The spectrum includes two massless vector multiplets corresponding to A^2_μ and A^3_μ and an extra chiral multiplet whose scalar field is ρ from the $N=2$ vector multiplet sector.

Models with $N=0$ also exist and can be studied by writing the full $N=2$ potential. They can be also obtained by further gauging the $N=2$ theory obtained in section 4.1, or by adding a superpotential to the $N=1$ theory. A complete description of the non supersymmetric phases will be done elsewhere. In this paper we will study the $N=0$ vacua which have vector charge with $g_0,g_1\neq0$ or with all $g_\Lambda\neq0$.

5. Non supersymmetric vacua

The study of $N=0$ vacua requires the knowledge of the potential of the scalar fields, which can be computed, for an abelian gauging, with the formula [54]

$$V=4h_{uv}k^u_\Lambda k^v_\Sigma L^\Lambda\bar{L}^\Sigma+\left(U^{\Lambda\Sigma}-3\bar{L}^\Lambda L^\Sigma\right)P^x_\Lambda P^x_\Sigma\,,$$

where

$$U^{\Lambda\Sigma}=-\frac{1}{2}(\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma})^{-1}-\bar{L}^\Lambda L^\Sigma,\qquad L^\Lambda=\mathrm{e}^{\frac{K}{2}}X^\Lambda=\mathrm{e}^{\frac{\tilde{K}}{2}}\mathrm{e}^{\frac{\hat{K}}{2}}X^\Lambda\,,$$

and

$$\tilde{K} = -\ln i(\rho - \bar{\rho})\,, \qquad \hat{K} = -\ln \frac{1}{2} i(\bar{\tau} - \tau) i(\bar{\sigma} - \sigma)\,.$$

The three contributions are the square of the supersymmetry variations of the hyperinos, gauginos and gravitinos respectively. The first two terms are positive definite while the last contribution is negative definite.

In the model at hand the last two terms become

$$\left(U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma\right) P^x_\Lambda P^x_\Sigma = -\mathrm{e}^{\tilde{K}} \eta^{\Lambda\Sigma} P^x_\Lambda P^x_\Sigma\,,$$

with $\tilde{K} = -\ln i(\rho - \bar{\rho})$. Then the scalar potential becomes

$$\begin{aligned} V &= 4h_{uv} k^u_\Lambda k^v_\Sigma L^\Lambda \bar{L}^\Sigma - \mathrm{e}^{\tilde{K}} \eta^{\Lambda\Sigma} P^x_\Lambda P^x_\Sigma \\ &= 4h_{uv} k^u_\Lambda k^v_\Sigma L^\Lambda \bar{L}^\Sigma + \mathrm{e}^{\tilde{K}} \left(P_2^2 + P_3^2\right) - \mathrm{e}^{\tilde{K}} \left(P_0^2 + P_1^2\right) \\ &= \mathrm{e}^{\tilde{K}} \left[4h_{uv} k^u_\Lambda k^v_\Sigma \mathrm{e}^{\hat{K}} X^\Lambda \bar{X}^\Sigma + P_2^2 + P_3^2 - P_0^2 - P_1^2\right], \end{aligned}$$

where we have used equation (2.16). Note that $V \geq 0$ if $P_0 = P_1 = 0$. In this case, all vacua with zero vacuum energy have unbroken $N = 2$ supersymmetry.

The potential can be computed by recalling that in the case of gauged axion isometries of the quaternionic manifold we have

$$P^x_\Lambda = \omega^x_u k^u_\Lambda\,.$$

We will consider here only the case when the C^m become charged under the $\Lambda = 0, 1$ symmetries.

Recalling equations (2.12) and (2.14), we see that the quaternionic metric h_{uv} along the C^m axion directions is

$$h_{mn} = \mathrm{e}^{2\phi} \left(L^{-1}\right)^I_m \left(L^{-1}\right)^I_n = \mathrm{e}^{2\phi}(\delta_{mn} + 2e^a_m e^a_n)\,,$$

with $I = 1, \ldots, 22$, $m, n = 1, 2, 3$, $a = 1, \ldots, 19$, while

$$\omega^x_m \omega^x_n = 2\mathrm{e}^{2\phi} \left(L^{-1}\right)^x_m \left(L^{-1}\right)^x_n = 2\mathrm{e}^{2\phi}(\delta_{mn} + e^a_m e^a_n)\,, \qquad x = 1, 2, 3.$$

Therefore, the scalar potential is

$$V = \mathrm{e}^{2\phi} \mathrm{e}^{\tilde{K}} \left[4(\delta_{mn} + 2e^a_m e^a_n) k^m_\Lambda k^n_\Sigma \mathrm{e}^{\hat{K}} X^\Lambda \bar{X}^\Sigma - 2(\delta_{mn} + e^a_m e^a_n) k^m_\Lambda k^n_\Sigma \eta^{\Lambda\Sigma}\right].$$

By taking $k^1_0 = g_0$ and $k^2_1 = g_1$ we have

$$\begin{aligned} V = \mathrm{e}^{2\phi} \mathrm{e}^{\tilde{K}} \Big[& 4\mathrm{e}^{\hat{K}} \Big(g_0^2 X^0 \bar{X}^0 + g_1^2 X^1 \bar{X}^1 + g_0 g_1 \Big(X^0 \bar{X}^1 + \bar{X}^0 X^1\Big) + \\ &+ 2e^a_1 e^a_1 g_0^2 X^0 \bar{X}^0 + 2e^a_2 e^a_2 g_1^2 X^1 \bar{X}^1 + 2e^a_1 e^a_2 g_0 g_1 \Big(X^0 \bar{X}^1 + \bar{X}^0 X^1\Big)\Big) - \\ &- 2\Big(g_0^2 + g_1^2 + g_0^2 e^a_1 e^a_1 + g_1^2 e^a_2 e^a_2\Big)\Big]\,. \end{aligned} \tag{5.1}$$

We want now to compute the extrema of the potential. The conditions

$$\frac{\partial V}{\partial \rho} = 0\,, \qquad \frac{\partial V}{\partial \phi} = 0$$

are satisfied at the points where $V = 0$. We will see that this is implied by the other extremum conditions. The equations

$$\frac{\partial V}{\partial \sigma} = 0\,, \qquad \frac{\partial V}{\partial \tau} = 0$$

are solved by

$$\mathcal{D}_\sigma X^0 = \mathcal{D}_\tau X^0 = \mathcal{D}_\sigma X^1 = \mathcal{D}_\tau X^1 = 0,$$

which gives

$$\sigma = \tau = i\,, \qquad \mathrm{e}^{\hat{K}} = \frac{1}{2}\,, \qquad X^0 = 1\,, \quad X^1 = -i\,, \quad X^2 = X^3 = 0\,,$$

so that $X^0\bar{X}^0 = X^1\bar{X}^1 = 1$. The conditions

$$\frac{\partial V}{\partial e_1^a} = 0\,, \qquad \frac{\partial V}{\partial e_2^a} = 0\,,$$

are fulfilled by $e_1^a = e_2^a = 0$.

As a function of σ and τ, the potential V is composed of two pieces $V = A(\sigma, \tau) + B$. $A(\sigma, \tau)$ is given by the two first lines in (5.1), and it is positive definite. B is negative and constant. The point $\sigma = \tau = i$ is an extremum of V and a minimum of A. The value of the potential at this extremum is

$$V(\sigma = \tau = i) = 2\mathrm{e}^{2\phi}\mathrm{e}^{\tilde{K}}\left(g_0^2 e_1^a e_1^a + g_1^2 e_2^a e_2^a\right) \geq 0\,.$$

This implies that the potential is positive definite for all σ and τ.

Summarizing, we have that $V \geq 0$ and that $V = 0$ at the extrema, so they are minima. The extremum condition does not fix the scalars ϕ, ρ, e_3^a and the remaining C^I (all of them except for C^m with $m = 1, 2$, which disappear from the spectrum.)

The gravitino mass matrix is

$$\begin{aligned} M_{AB} &= 2S_{AB} = iL^\Lambda P_\Lambda^x(\sigma^x)_{AB} = i\mathrm{e}^\phi \begin{pmatrix} L^0 g_0 + iL^1 g_1 & 0 \\ 0 & -L^0 g_0 + iL^1 g_1 \end{pmatrix} \\ &= \mathrm{e}^\phi \mathrm{e}^{\frac{\hat{K}}{2}} \frac{1}{[i(\rho - \bar{\rho})]^{\frac{1}{2}}} \begin{pmatrix} X^0 g_0 + iX^1 g_1 & 0 \\ 0 & -X^0 g_0 + iX^1 g_1 \end{pmatrix}. \end{aligned}$$

At the point $\sigma = \tau = i$ we have $X^0 = 1$ and $X^1 = -i$, so that the eigenvalues squared are:

$$m_{1,2}^2 = \frac{1}{2}\frac{\mathrm{e}^{2\phi}}{i(\rho - \bar{\rho})}(g_0 \pm g_1)^2\,,$$

which gives the gravitino masses measured in terms of the **K3** volume and the $\mathbf{T}^2$ Kähler modulus.

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5.1 More general vacua

More general vacua, preserving $N = 1, 0$ supersymmetry, can be obtained by considering an arbitrary vector coupling g_Λ. g_0 and g_1 gauge two of the isometries C^m, while g_2 and g_3 gauge two of the isometries C^a.

Taking the Killing vectors as

$$k_0^{C^{m=1}} = g_0\,, \quad k_1^{C^{m=2}} = g_1\,, \quad k_2^{C^{a=1}} = g_2\,, \quad k_3^{C^{a=2}} = g_3\,,$$

we first notice that the condition on the vector multiplet sector $X^2 = X^3 = 0 \Longrightarrow \sigma = \tau = i$ still holds. In the hypermultiplet sector, the condition for $N = 1$ vacua we had, $e^a_{m=1,2} = 0$, is supplemented by the extra condition, coming from the gauginos variation, $e_m^{a=1,2} = 0$. This eliminates from the spectrum two extra scalars $e_3^{a=1,2} = 0$ together with the axions $C_{a=1,2}$. We are therefore left 18 chiral multiplets from the hypermultiplet sector, no massless vector multiplets and one chiral multiplet from the $N = 2$ vector multiplet sector.

If we relax the condition $|g_0| = |g_1|$, the vacua will not preserve any supersymmetry, but still will have vanishing vacuum energy, as can be shown by looking at the scalar potential.

We want to note the close connection of the present $N = 2$ model with another $N = 2$ model more recently discussed as an effective theory for $N = 2$ vacua of the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold [25]. The vector multiplet sector of that theory is obtained by "higgsing" two of the three vector multiplets without breaking $N = 2$ supersymmetry, as discussed in section 4.1. The scalars in the remaining hypermultiplets parametrize the manifold $\mathrm{SO}(4,18)/(\mathrm{SO}(4) \times \mathrm{SO}(18))$, but since the vector multiplet sector is the the same as in the $T^6/\mathbb{Z}_2$ truncated model of ref. [25], the pattern of the supersymmetry breaking is very similar, and is insensitive to the number of the hypermultiplets. This is because the relation

$$P^x_\Lambda P^x_\Sigma \left(U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma\right) = -2P^x_\Lambda P^x_\Sigma \bar{L}^\Lambda L^\Sigma$$

is still satisfied as in the model of ref. [25].

This model was also shown to be connected to the minimal $N = 2$ model studied in refs. [34, 35]. The vanishing potential of the theory in ref. [25] was closely connected to the positive potential of the theory in refs. [34, 35], which lead to moduli stabilization, as expected from the $\mathbf{T}^6/\mathbb{Z}_2$ orientifold analysis [7, 8, 9, 56, 57, 27].

5.2 $N = 1 \to N = 0$ no scale supergravities

In this section we find, by truncation from $N = 2$, a $N = 1$ theory with a transition $N = 1 \to N = 0$ [58]. The truncation $N = 2 \to N = 1$ can be formally obtained by integrating out the second gravitino multiplet, together with the states which receive mass in the $N = 2 \to N = 1$ phase transition. Twenty chiral multiplets from the hypermultiplet sector remain massless in the truncation together with one chiral multiplet from the vector multiplet sector.

On the other hand, by relaxing the condition $|g_0| = |g_1|$, which makes the transition $N = 1 \to N = 0$, none of the scalars in the $N = 1$ theory take mass. This means that the $N = 1 \to N = 0$ transition must occur with a vanishing potential. To understand this, we analyze the moduli space of the 21 chiral multiplets. The left over moduli from the quaternionic manifold parametrize the submanifold

$$\frac{\mathrm{SO}(2,20)}{\mathrm{SO}(2)\times\mathrm{SO}(20)}\,,$$

which is a Kähler-Hodge manifold with Kähler potential

$$K_1 = -\ln\left[(x_0+\bar{x}_0)^2 - \sum_{a=1}^{19}(x_a+\bar{x}_a)^2\right], \tag{5.2}$$

where

$$x^a = \frac{1}{2}(e_3^a + iC^a)\,, \qquad x_0 = \frac{1}{2}(\mathrm{e}^{-\phi} + iC^{m=3})\,,$$

and (ϕ, e_3^a) parametrize the moduli space of the metrics

$$\frac{\mathrm{SO}(1,19)}{\mathrm{SO}(19)} \times \mathbb{R}^+_{\mathbf{T}^2}\,.$$

The remaining chiral multiplet contains the **K3** volume modulus with Kähler potential $\tilde{K} = -\ln i(\rho-\bar{\rho})$. The total Kähler potential of the manifold of the scalars in the chiral multiplets is a cubic polynomial

$$K = -\ln i(\rho-\bar{\rho})\left[(x_0+\bar{x}_0)^2 - \sum_{a=1}^{19}(x_a+\bar{x}_a)^2\right].$$

The flux which breaks $N=1$ to $N=0$ corresponds to a constant superpotential $W = a =$ constant. In this situation the scalar potential [59],

$$V = \mathrm{e}^K\left(\mathcal{D}_i W \mathcal{D}_{\bar{j}}\bar{W}G^{i\bar{j}} - 3|W|^2\right), \qquad G^{i\bar{j}} = \left(\partial_i\partial_{\bar{j}}K\right)^{-1}, \quad \mathcal{D}_i W = (\partial_i + \partial_i K)W\,,$$

is identically zero [31, 33, 60]. The gravitino mass is related to the overall volume of $\mathbf{K3}\times\mathbf{T}^2$,

$$m_{3/2}^2 = \mathrm{e}^K a^2\,.$$

This is a standard $N=1$ no scale model. However, it is different from the one obtained by $\mathbf{T}^6/\mathbb{Z}_2$ compactification because it has a much richer structure of moduli. The goldstino is essentially the fermion superpartner of the overall $\mathbf{K3}\times\mathbf{T}^2$ volume while all the other fermions receive a mass due to the flux, equal to the gravitino mass [31].

Note that if instead of twenty chiral multiplets we had considered a model with two chiral multiplets in (5.2), we would have retrieved the analysis of [58].

6. Concluding remarks

In the present investigation we have shown that compactifications on $\mathbf{K3}\times\mathbf{T}^2/\mathbb{Z}_2$ orientifold can be reproduced by a gauged $N=2$ supergravity which exactly gives the same $N=2,1,0$ vacua as obtained by analyzing the existence of the ten dimensional supergravity solution.

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The choice of the gauging is the crucial issue. The existence of backgrounds with vanishing vacuum energy and broken supergravity closely depends on the fact that the corresponding gauging requires a choice of symplectic sections for special geometry which do not admit a prepotential. This is required in order to evade a no-go theorem on partial breaking of supersymmetry [62].

This analysis can be generalized by including Yang-Mills degrees of freedom coming from the branes, as well as more general fluxes related to the supergravity charges $f_{m\Lambda}$, $h_{a\Lambda}$.

Our analysis extends previous studies on partial super-Higgs in $N = 2$ supergravity considered in the literature [33, 63, 64, 61]. In particular the no-scale structure is closely related to the minimal model [34, 35] and it only depends on universal properties leading to cancellation of positive and negative contributions in the scalar potential as it occurred in $N = 1$ no-scale models [31, 32].

Another interesting problem which is left aside here, is the effect of the quantum corrections in these no-scale models. Some work along these directions has recently appeared in the literature [28, 65].

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